

The Trispectrum in the Multi-brid Inflation

Qing-Guo Huang *

School of Physics, Korea Institute for Advanced Study, 207-43, Cheongryangri-Dong,
Dongdaemun-Gu, Seoul 130-722, Korea

ABSTRACT: The trispectrum is at least as important as the bispectrum and its size can be characterized by two parameters τ_{NL} and g_{NL} . In this short paper, we focus on the Multi-brid inflation, in particular the two-brid inflation model in arXiv.0805.0974, and find that τ_{NL} is always positive and roughly equals to $(\frac{6}{5}f_{NL})^2$ for the low scale inflation, but g_{NL} can be negative or positive and its order of magnitude can be the same as that of τ_{NL} or even larger.

KEYWORDS: Trispectrum, Multi-brid inflation.

*huangqg@kias.re.kr

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1. Introduction

The single-field slow-roll inflation predicts a roughly Gaussian distribution of the primordial power spectrum [1]. However in the general case with a large number of fields, a large deviation from Gaussian distribution is expected. A well-understood ansatz of non-Gaussianity has a local shape. Working in the framework of Fourier transformation of ζ , the primordial power spectrum \mathcal{P}_ζ is defined by

$$\langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \rangle = (2\pi)^3 \mathcal{P}_\zeta(k_1) \delta^3(\mathbf{k}_1 + \mathbf{k}_2), \quad (1.1)$$

and the primordial bispectrum and trispectrum are defined by

$$\langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3), \quad (1.2)$$

$$\langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \zeta(\mathbf{k}_4) \rangle = (2\pi)^3 T_\zeta(k_1, k_2, k_3, k_4) \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4). \quad (1.3)$$

The bispectrum and trispectrum are respectively related to the power spectrum by

$$B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{NL} [\mathcal{P}_\zeta(k_1) \mathcal{P}_\zeta(k_2) + 2 \text{ perms}], \quad (1.4)$$

$$\begin{aligned} T_\zeta(k_1, k_2, k_3, k_4) = & \tau_{NL} [\mathcal{P}_\zeta(k_1) \mathcal{P}_\zeta(k_2) \mathcal{P}_\zeta(k_3) + 11 \text{ perms}] \\ & + \frac{54}{25} g_{NL} [\mathcal{P}_\zeta(k_1) \mathcal{P}_\zeta(k_2) \mathcal{P}_\zeta(k_3) + 3 \text{ perms}], \end{aligned} \quad (1.5)$$

where f_{NL} , τ_{NL} and g_{NL} are the non-Gaussianity parameters which measure the size of the non-Gaussianity.

In the δN formalism [2], the curvature perturbation in the multi-field inflation model can be expanded as

$$\begin{aligned} \zeta = \delta N = & N(\phi_i + \delta\phi_i) - N(\phi_i) \\ = & N_{,i} \delta\phi_i + \frac{1}{2} N_{,ij} \delta\phi_i \delta\phi_j + \frac{1}{6} N_{,ijk} \delta\phi_i \delta\phi_j \delta\phi_k + \dots, \end{aligned} \quad (1.6)$$

where

$$N_{,i} = \frac{\partial N}{\partial \phi_i}, \quad N_{,ij} = \frac{\partial^2 N}{\partial \phi_i \partial \phi_j}, \quad N_{,ijk} = \frac{\partial^3 N}{\partial \phi_i \partial \phi_j \partial \phi_k}, \quad (1.7)$$

and the repeated sub-indices are summed over. Therefore the non-Gaussianity parameters are given by, [3],

$$f_{NL} = \frac{5}{6} \frac{N_{,ij} N_{,i} N_{,j}}{(N_{,l} N_{,l})^2}, \quad (1.8)$$

$$\tau_{NL} = \frac{N_{,ij} N_{,ik} N_{,j} N_{,k}}{(N_{,l} N_{,l})^3}, \quad (1.9)$$

$$g_{NL} = \frac{25}{54} \frac{N_{,ijk} N_{,i} N_{,j} N_{,k}}{(N_{,l} N_{,l})^3}. \quad (1.10)$$

From the Cauchy-Schwarz inequality, the τ_{NL} has a lower bound, [4],

$$\tau_{NL} \geq \left(\frac{6}{5} f_{NL}\right)^2. \quad (1.11)$$

The above inequality is saturated in the single field case, or the vector $N_{,i}$ is an eigenvector of the matrix $N_{,ij}$ in the case with multi fields. So τ_{NL} is expected to be large if $f_{NL} \gg 1$. Since $N_{,ijk}$ is quite model-dependent, g_{NL} can be negative or positive, and its order of magnitude can be large or small.

WMAP 5yr data [5] implies

$$-9 < f_{NL}^{local} < 111 \quad (1.12)$$

at 2σ level. Even though the Gaussian distribution is still consistent with data, the allowed negative part of f_{NL}^{local} has been cut from the WMAP 3yr data significantly. For example, if $\tau_{NL} > 560$, it can also be detected by Planck [6]. In [6] the authors also pointed out that the trispectrum of the Planck data is more sensitive to primordial non-Gaussianity than the bispectrum for $|f_{NL}| \gtrsim 50$. The trispectrum is at least as important as the bispectrum.

As we know, a large local-shape f_{NL} can be achieved in the multi-brid inflation. In this short paper, we investigate the trispectrum in the special two-brid inflation model in [7]. One can discuss more complicated setups, such as [8]. However, we will see that the phenomenology in this simple model is rich enough. In Sec.2, we calculate the trispectrum in this two-brid inflation model and discuss its property in detail. Some discussions are given in Sec. 3.

2. The trispectrum in the Multi-brid inflation model

In this section, we consider the two-brid inflation in [7] with the potential

$$V(\phi_1, \phi_2) = V_0 \exp(\alpha_1 \phi_1 + \alpha_2 \phi_2), \quad (2.1)$$

where α_1 and α_2 are two free parameters in the unit of $M_p = 1$. The effective mass for each inflaton is $m_i = \sqrt{3}\alpha_i H_*$ where H_* is the Hubble parameter during inflation. Here

the inflatons are assumed to slowly roll down their potentials and the slow-roll conditions require $\alpha_i \ll 1$ for $i = 1, 2$. The dynamics of these two inflatons is governed by

$$\frac{d\phi_i}{dN} \simeq \alpha_i, \quad (i = 1, 2), \quad (2.2)$$

where $dN = -Hdt$. In [7], V_0 is prompted to be

$$V_0 = \frac{1}{2} \sum_{i=1}^2 g_i^2 \phi_i^2 \chi^2 + \frac{\lambda}{4} (\chi^2 - \frac{\sigma^2}{\lambda})^2. \quad (2.3)$$

Therefore the inflation ends when $\phi_i = \phi_{i,f}$ which is given by

$$\sum_{i=1}^2 g_i^2 \phi_{i,f}^2 = \sigma^2. \quad (2.4)$$

For convenience, we parametrize $\phi_{i,f}$ as

$$\phi_{1,f} = \frac{\sigma}{g_1} \cos \gamma, \quad \phi_{2,f} = \frac{\sigma}{g_2} \sin \gamma. \quad (2.5)$$

Without loss of generality, we assume $\alpha_i \phi_i > 0$ (for $i = 1, 2$).

The number of e-folds before the end of inflation is given by

$$N = \frac{1}{2} \ln \left(\frac{e^{2\phi_1/\alpha_1} + e^{2\phi_2/\alpha_2}}{e^{2\phi_{1,f}/\alpha_1} + e^{2\phi_{2,f}/\alpha_2}} \right). \quad (2.6)$$

Because the surface where the inflation ends in the field space is not exactly the surface of constant energy density, the author in [7] introduced a correction to the number of e-folds before the end of inflation as

$$N_c = \frac{\sigma}{4} \left(\frac{\alpha_1}{g_1} \cos \gamma + \frac{\alpha_2}{g_2} \sin \gamma \right). \quad (2.7)$$

However, this correction term can be neglected if $\alpha_i \ll 1$. So we ignore it in our paper. In order to simplify the notation, we define some new parameters as

$$g = \sqrt{g_1^2 \cos^2 \gamma + g_2^2 \sin^2 \gamma}, \quad (2.8)$$

$$\alpha_1 = \alpha \cos \theta, \quad \alpha_2 = \alpha \sin \theta. \quad (2.9)$$

We expand δN to the third order,

$$\begin{aligned} \delta N = & \frac{g_1 \cos \gamma \delta \phi_1 + g_2 \sin \gamma \delta \phi_2}{\alpha g c_+} + \frac{g_1^2 g_2^2 (\sin \theta \delta \phi_1 - \cos \theta \delta \phi_2)^2}{2\sigma g^3 \alpha c_+^3} \\ & - \frac{g_1^3 g_2^3}{2\sigma^2 g^4} \frac{c_- (\sin \theta \delta \phi_1 - \cos \theta \delta \phi_2)^3}{\alpha c_+^5}, \end{aligned} \quad (2.10)$$

where

$$c_- = \frac{g_1}{g} \cos \theta \sin \gamma - \frac{g_2}{g} \sin \theta \cos \gamma, \quad (2.11)$$

$$c_+ = \frac{g_1}{g} \cos \theta \cos \gamma + \frac{g_2}{g} \sin \theta \sin \gamma. \quad (2.12)$$

Assume that the scalar field fluctuations $\delta\phi_1$ and $\delta\phi_2$ are Gaussian and non-correlated,

$$\langle \delta\phi_i \delta\phi_j \rangle = \left(\frac{H_*}{2\pi} \right)^2 \delta_{ij}, \quad (2.13)$$

where H_* denotes the Hubble parameter at the time of Hubble exit. Hence the amplitude of the primordial power spectrum is

$$P_\zeta = \frac{1}{\alpha^2 c_+^2} \left(\frac{H_*}{2\pi} \right)^2. \quad (2.14)$$

Since the amplitude of the tensor perturbation is still given by

$$P_T = 8 \left(\frac{H_*}{2\pi} \right)^2, \quad (2.15)$$

the tensor-scalar ratio becomes

$$r = P_T/P_\zeta = 8\alpha^2 c_+^2. \quad (2.16)$$

We can also easily calculate the spectral index, namely

$$n_s = 1 - \alpha^2. \quad (2.17)$$

The spectral index and tensor scalar ratio can be measured by the experiments. Once they are fixed, the parameters α and c_+ in this two-brid inflation model are fixed as well. WMAP 5yr data [5] implies

$$n_s = 0.96^{+0.014}_{-0.013}. \quad (2.18)$$

For $n_s = 0.96$, $\alpha = 0.2$. On the other hand, we can prove that $c_+^2 \leq 1$ which implies

$$r \leq 8(1 - n_s), \quad (2.19)$$

where the equality is satisfied when

$$g_1 \sin \theta \cos \gamma = g_2 \cos \theta \sin \gamma. \quad (2.20)$$

Eq.(2.19) can be taken as a consistency relation for this two-brid inflation model. For $n_s = 0.96$, $r \leq 0.32$ which is consistent with WMAP 5yr data ($r < 0.20$).

From Eqs.(1.4) and (1.5), the non-Gaussiniaty parameters are given by

$$f_{NL} = \frac{5\alpha g_1^2 g_2^2}{6\sigma g^3} \frac{\tilde{c}^2}{c_+}, \quad (2.21)$$

$$\tau_{NL} = \frac{\alpha^2 g_1^4 g_2^4}{\sigma^2 g^6} \frac{\tilde{c}^2}{c_+^2}, \quad (2.22)$$

$$g_{NL} = \frac{25\alpha^2 g_1^3 g_2^3}{18\sigma^2 g^4} \frac{c_- \tilde{c}^3}{c_+^2}, \quad (2.23)$$

where

$$\tilde{c} = \frac{g_2}{g} \cos \theta \sin \gamma - \frac{g_1}{g} \sin \theta \cos \gamma. \quad (2.24)$$

We see that the second order non-Gaussianity parameters are related to f_{NL} by

$$\tau_{NL} = \frac{36}{25} \frac{1}{\tilde{c}^2} f_{NL}^2, \quad (2.25)$$

$$g_{NL} = \frac{2g^2}{g_1 g_2} \frac{c_-}{\tilde{c}} f_{NL}^2. \quad (2.26)$$

Since

$$\tilde{c}^2 = 1 - c_+^2 \quad (2.27)$$

and $c_+^2 \leq 1$, $\tilde{c}^2 \leq 1$ and thus

$$\tau_{NL} \geq \left(\frac{6}{5} f_{NL}\right)^2. \quad (2.28)$$

The inequality is saturated when

$$g_1 \cos \theta \cos \gamma = -g_2 \sin \theta \sin \gamma. \quad (2.29)$$

Keeping f_{NL} fixed, τ_{NL} can be much larger than f_{NL}^2 if $c_+ \rightarrow 1$. However, in the limit of $c_+ \rightarrow 1$, f_{NL} should approach to 0. Here we want to remind the readers that both α and c_+ can be fixed by the spectral index and the tensor-scalar ratio. Considering

$$c_+^2 = \frac{r}{8(1 - n_s)}, \quad (2.30)$$

$c_+^2 < 0.625$ for $n_s = 0.96$ and $r < 0.20$. If $r \ll 1$, $c_+ \sim 0$ and then $\tau_{NL} \simeq \frac{36}{25} f_{NL}^2$.

2.1 $g_1 = g_2$

From Eq.(2.8), $g_1 = g_2 = g$ in this case. Now c_- , \tilde{c} , and c_+ are simplified to be

$$c_- = \tilde{c} = \sin(\gamma - \theta), \quad c_+ = \cos(\gamma - \theta). \quad (2.31)$$

Therefore

$$f_{NL} = \frac{5\alpha g}{6\sigma} \left(\frac{1}{c_+} - c_+ \right), \quad (2.32)$$

$$\tau_{NL} = \frac{36}{25(1 - c_+^2)} f_{NL}^2, \quad (2.33)$$

$$g_{NL} = 2f_{NL}^2. \quad (2.34)$$

In this special case, g_{NL} is positive and is related to f_{NL} by $2f_{NL}^2$. If $r \ll 1$, $c_+ \sim 0$ and then $\tau_{NL} \simeq \frac{36}{25} f_{NL}^2$ which is roughly the same order of magnitude as g_{NL} , and

$$f_{NL} \simeq \frac{5\alpha g}{6\sigma c_+}. \quad (2.35)$$

Because $\chi = 0$ during inflation, the Hubble parameter is related to σ by

$$H_*^2 = \frac{\sigma^4}{12\lambda}. \quad (2.36)$$

Considering $H_* = 1.1 \times 10^{-4} r^{\frac{1}{2}}$, σ is related to r by

$$\sigma = 1.95 \times 10^{-2} \lambda^{\frac{1}{4}} r^{\frac{1}{4}}. \quad (2.37)$$

If $r \ll 1$, the non-Gaussianity parameter f_{NL} takes the form

$$f_{NL} \simeq 121(1 - n_s) \frac{g}{r} \left(\frac{r}{\lambda}\right)^{\frac{1}{4}}. \quad (2.38)$$

On the other hand, σ is required to be large compared to the Hubble parameter during inflation for the field χ to work as a water-fall field. So we have

$$(r/\lambda)^{\frac{1}{4}} \lesssim 177. \quad (2.39)$$

Therefore

$$f_{NL} \lesssim 2.1 \times 10^4 (1 - n_s) \frac{g}{r}. \quad (2.40)$$

For $n_s = 0.96$, $f_{NL} \lesssim 810g/r$.

To summarize, if $g_1 = g_2$, f_{NL} is positive and can be very large, and $g_{NL} = 2f_{NL}^2$. In this case, $\tau_{NL} \geq \frac{36}{25}f_{NL}^2$ and the equality is roughly satisfied if $r \ll 1$.

2.2 $g_1 \neq g_2$

If $g_1 = g_2$, we find that $g_{NL} = 2f_{NL}^2$ which must be positive. In this subsection, we focus on the case of $g_1 \neq g_2$ and investigate whether g_{NL} can be negative, but the absolute value is still large. Since there is a symmetry between ϕ_1 and ϕ_2 , we assume

$$\Delta = \frac{g_1}{g_2} < 1, \quad (2.41)$$

without loss of generality.

Keeping f_{NL} fixed, a large absolute value of g_{NL} might be obtained if $\tilde{c} = 0$ which implies

$$\gamma = \gamma_0 = \tan^{-1}(\Delta \tan \theta). \quad (2.42)$$

On the other hand, f_{NL} should be 0 when $\tilde{c} = 0$. So we consider that γ slightly deviates from γ_0 , namely

$$\gamma = \gamma_0 + \delta, \quad (2.43)$$

with $\delta \ll \gamma_0$. For $\Delta \ll 1$, we have

$$g \simeq \frac{g_1}{\kappa}, \quad (2.44)$$

$$\tilde{c} \simeq \frac{\kappa^2}{\Delta} \delta, \quad (2.45)$$

$$c_- \simeq -\frac{\sin 2\theta}{2\Delta}, \quad (2.46)$$

$$c_+ \simeq 1, \quad (2.47)$$

where

$$\kappa = \sqrt{\cos^2 \theta + \Delta^2 \sin^2 \theta}. \quad (2.48)$$

In this limit, $c_+ \simeq 1$ and then the tensor-scalar ratio is slightly larger than the WMAP bound if $n_s = 0.96$. Here we ignore this constraint first and illustrate an important observation. Now we have

$$f_{NL} \simeq \frac{5\alpha g_2}{6\sigma} \frac{\kappa^7}{\Delta^3} \delta^2, \quad (2.49)$$

and

$$\tau_{NL} \simeq \frac{36}{25} \frac{\Delta^2}{\kappa^4 \delta^2} f_{NL}^2, \quad (2.50)$$

$$g_{NL} \simeq -\frac{\Delta \sin 2\theta}{\kappa^4 \delta} f_{NL}^2. \quad (2.51)$$

Since both Δ and $\sin 2\theta$ are positive, g_{NL} can be negative if $\delta > 0$. In order to get the precise results and go beyond the above approximation, we should adopt the numerical calculation. For example, inputting $n_s = 0.96$, $r = 0.22$, $\Delta = 0.1$, we have

$$\tau_{NL}/f_{NL}^2 \simeq 4.6, \quad (2.52)$$

and

$$\theta = 0.79, \gamma = 0.466 \rightarrow f_{NL} = 0.03 \frac{\alpha g_2}{\sigma}, g_{NL}/f_{NL}^2 \simeq -10; \quad (2.53)$$

$$\theta = 1.38, \gamma = 0.1 \rightarrow f_{NL} = 1.12 \frac{\alpha g_2}{\sigma}, g_{NL}/f_{NL}^2 \simeq 4.92. \quad (2.54)$$

So there is still a big room for a large f_{NL} . We see that a negative g_{NL} with large absolute value can be obtained in the case of $g_1 \neq g_2$.

3. Conclusions

The non-Gaussianity is expected to be large in the multi-brid inflation. In the simple two-brid inflation model [7], τ_{NL} must be positive and roughly equals to $(\frac{6}{5}f_{NL})^2$ due to the upper bound on the inflation scale ($r < 0.2$), but g_{NL} can be negative or positive and its order of magnitude can be the same as that of τ_{NL} or even larger. In [9] the authors consider a more complicated case where τ_{NL} can be much larger than $(\frac{6}{5}f_{NL})^2$, but it seems unlikely for the case with red-tilted power spectrum and large positive f_{NL} .

As we know, multi-brid inflation model and curvaton model are the only cases that can be analyzed systematically (over a fairly wide range of the model parameters) and are capable of yielding fairly large local-shape f_{NL}^{local} . In the usual single curvaton model, the curvature perturbation is assumed to be generated by curvaton and then we have $\tau_{NL} = (\frac{6}{5}f_{NL})^2$. The size of g_{NL} depends on the curvaton potential. If the relevant part of curvaton potential takes the exactly quadratic form, we have $g_{NL} = -\frac{10}{3}f_{NL}$. If a non-quadratic correction becomes visible compared to the quadratic term, g_{NL} in the curvaton model can have a large deviation from the relation $g_{NL} = -\frac{10}{3}f_{NL}$ and the sign of g_{NL} depends on the sign of the non-quadratic term which can be positive or negative as well. The order of magnitude of g_{NL} for the curvaton model with non-quadratic correction can be roughly the same as f_{NL}^2 which takes the same order of τ_{NL} . See [10] in detail. If the

curvaton potential is dominated by the non-quadratic term, $\tau_{NL} \sim g_{NL} \sim f_{NL}^2$ [11]. On the other hand, in the mixed curvaton scenario [11, 12] where the total curvature perturbation is still mainly produced by the inflaton, not curvaton, τ_{NL} is enhanced by a large factor $1/\beta$ with respect to $(\frac{6}{5}f_{NL})^2$, where β measures the size of curvature perturbation caused by curvaton compared to the total curvature perturbation including the contribution from inflaton. So τ_{NL} can be much larger than $(\frac{6}{5}f_{NL})^2$ in the mixed curvaton scenario. This is different from the multi-brid inflation. In the mixed curvaton model, whether g_{NL} is enhanced or suppressed depends on how g_{NL}^{cv} is related to f_{NL}^{cv} , where the superscript “cv” denotes the non-Gaussianity generated by curvaton in the usual curvaton model. For example, if $g_{NL}^{cv} \sim f_{NL}^{cv}$, $g_{NL} \sim \beta f_{NL}$ which is expected to be very small and undetectable, but if $g_{NL}^{cv} \sim (f_{NL}^{cv})^2$, $g_{NL} \sim (f_{NL})^2/\beta$ which is enhanced and may be detectable. If we can not only detect f_{NL} , but also get some restricted constraints on τ_{NL} and g_{NL} in the forthcoming experiments, it is possible to distinguish multi-brid inflation model from curvaton model.

Recently there are many papers concerning on the bispectrum and the trispectrum [13, 14]. Here we want to stress that the trispectrum might be as important as the bispectrum and we encourage more theorists and experimenters to pay more attentions to the trispectrum in the near future.

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